

Noether And Some Other Dynamical Symmetries In Kantowski-Sachs Model

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Abstract

The forms of coupling of the scalar field with gravity, appearing in the induced theory of gravity, and the potential are found in the Kantowski-Sachs model under the assumption that the Lagrangian admits Noether symmetry. The form thus obtained makes the Lagrangian degenerate. The constrained dynamics thus evolved due to such degeneracy has been analysed and a solution has also been presented which is inflationary in behaviour. It has further been shown that there exists other technique to explore the dynamical symmetries of the Lagrangian and that is simply by inspecting the field equations. Through this method Noether along with some other dynamical symmetries are found which do not make the Lagrangian degenerate.

PACS 98.00.Dr.- Induced Gravity Theory, Noether Symmetry, Cosmology.

1 Introduction

The theory of induced gravity has been found to be a strong candidate in several unified theories [1], in which the Einstein-Hilbert action appears as an effective action induced by the quantum properties of the vacuum state of matter field in the weak energy limit. The beauty of the theory lies in the fact that it identifies the inflaton with the scalar field inducing the Newtonian gravitational constant (G_N) and the cosmological constant (Λ). The theory has also been found to overcome the shortcomings of the old inflationary theory viz. the graceful exit problem [2] and the longstanding problem of density perturbation [3]. Further it has been observed [4] that the theory also preserves the generic features of Vilenkin [5] and Hartle-Hawking [6] wave functions. Finally it has also been shown that [7] the theory admits wormhole solutions both for real and imaginary fields.

Despite such a wide range of successful applications of the theory, the actual form of coupling of the matter field with gravity ($f(\phi)$) is not known a priori. One usually chooses it in an ad hoc manner. Capozziello et al [8] made an attempt to find the form of such coupling under the assumption that the Lagrangian of the induced theory of gravity admits Noether symmetry, which further restricts the form of the potential for the scalar field. In the Robertson-Walker model they have observed that [8] $k \neq 0$ imposes strong constraints in the form of coupling and the potential. The form of coupling thus obtained makes the Lagrangian degenerate and the form of the potential was found to be sixth order in the scalar field ϕ , for which only trivial solution is admissible.

When the Hessian determinant $W = |\frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j}|$ vanishes the Lagrangian becomes degenerate which imposes a constraint in the sense that the Legendre transformation does not exist and hence the Hamiltonian of the system can not be defined unless such constraints are analysed properly. In the domain of Lagrangian dynamics the constraint implies more number of degrees of freedom than the number of field equations, which means, one has to make certain assumptions to obtain exact solutions. However such degeneracy does not in any way lead to trivial solutions. This has been pointed out in a recent communication [9], where it has been shown that the existence of only trivial solutions is not due to the presence of degeneracy in the Lagrangian, rather due to the existence of

Noether potential in the form $V(\phi) = \Lambda\phi^6$, which does not satisfy the field equations. This is a striking feature and perhaps not been encountered earlier. The reason for such contradiction, that the Nöther symmetry of the Lagrangian restricts the form of the potential in such a manner that it does not satisfy the field equations, is not known at present. However, it has been shown in the paper [9] that for a choice of the coupling parameter in the form $f(\phi) = \epsilon\phi^2$, $\epsilon \neq -1/12$, which does not make the Lagrangian degenerate and for a quartic potential the Lagrangian admits certain dynamical symmetries, other than Noether symmetry, along with a conserved current. We emphasize on the fact that the symmetry thus obtained can not be explored by the standard technique of finding dynamical symmetries via Noether theorem. Rather, it is found simply from a combination of the field equations. This puts forward a vital question on the longstanding claim that all the dynamical symmetries of a physical system are Noether symmetries.

Motivated by the above mentioned result, our attempt is now to find the form of coupling $f(\phi)$ in models other than Robertson-Walker, under the same assumption that the Lagrangian admits Noether symmetry. In the present paper the Kantowski-Sachs metric has been taken under consideration. In this model, once again, we observe that Noether symmetry exists at the cost of imposing degeneracy in the Lagrangian. However, the potential this time turns out to be quartic in the scalar field ϕ which satisfies the field equations, in contrast to the Robertson-Walker model. The constraint imposed by the degeneracy has been analysed in the domain of Lagrangian dynamics, which has been found to yield an excess number of the degrees of freedom to the field equations. However, the most interesting aspect of the present work is that, instead of working with the whole lengthy process of finding Noether symmetry, it has been shown to obtain the same, just by inspecting the field equations. In this method the coupling parameter has been chosen in the form $f(\phi) = \epsilon\phi^2$ as in the Robertson-Walker model [9]. It has been observed that the quartic form of the potential yields the Noether symmetry for $\epsilon = -1/12$. However, for any other arbitrary ϵ there exists yet another symmetry along with a conserved current, keeping the Lagrangian nondegenerate. This is surprising that such an inherent symmetry of the system can not be explored via Noether theorem. Hence we conclude that there exists dynamical symmetries of a system other than Noether symmetry. The symmetry thus obtained has got important consequences. Fakir, Unruh and Habib [3] have shown that large negative ϵ may lead to well behaved self-consistent classical solutions which admit inflationary behaviour. Further in the framework of chaotic inflation, it can also produce density perturbations of amplitude consistent with the large scale behaviour, keeping the cosmological constant Λ within the order (10^{-2}) of the ordinary GUT range. Since the symmetry that we have explored does not make any restriction on ϵ in general, so the results of Fakir, Unruh and Habib [3] should also be realised, in principle, in this case too. Finally, that the large negative value of ϵ admits wormhole solution has already been confirmed in [7]

This paper is organised in the following manner. In section 2, the field equations are obtained from the action principle and the condition for which the Hessian determinant vanishes yielding a degenerate Lagrangian has been found. The form of coupling $f(\phi)$, the potential $V(\phi)$ and the conserved current are then obtained by studying the Nöther symmetry. It has been found that for the existence of such symmetry, the Lagrangian turns out to be degenerate. In section 3, the constraint imposed by the degenerate system is analysed in the domain of Lagrangian dynamics, whose outcome is a pair of field equations in first order for three degrees of freedom. This implies that one has to make one physically reasonable assumption to obtain nontrivial solutions. A solution has also been presented at the end of this section. In section 4, it has been shown that the above mentioned symmetry could have been obtained quite easily just by inspecting the field equations. Further it has also been found that some other dynamical symmetry for the system still exists, that can not be obtained by applying the Noether theorem and that does not make the Lagrangian degenerate. Thus it is confirmed that not all the dynamical symmetries hidden in a Lagrangian could be obtained by the application of Noether's theorem. Concluding remarks are presented in section 5.

2 Noether Symmetry In Kantowski-Sachs Model

We start with the following action,

$$A = \int d^4X \sqrt{-g} [f(\phi)R - \frac{1}{2}\phi_{,\mu}\phi'^{\mu} - V(\phi)] \quad (1)$$

which for the Kantowski-sachs metric

$$ds^2 = -dt^2 + a^2dr^2 + b^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (2)$$

reduces to

$$A = 4\pi \int [-4f'abb\dot{\phi} - 2f'b^2\dot{a}\dot{\phi} - 4fb\dot{a}\dot{b} - 2fab^2 + 2fa + \frac{1}{2}ab^2\dot{\phi}^2 - ab^2V(\phi)]dt + \text{surface-term}. \quad (3)$$

Field equations are

$$2\frac{\ddot{b}}{b} + \frac{f'}{f}\ddot{\phi} + \frac{f''}{f}\dot{\phi}^2 + 2\frac{f'}{f}\frac{\dot{b}}{b}\dot{\phi} + \frac{\dot{b}^2}{b^2} + \frac{\dot{\phi}^2}{4f} + \frac{1}{b^2} - \frac{V(\phi)}{2f} = 0 \quad (4)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{f'}{f}\ddot{\phi} + \frac{\dot{a}\dot{b}}{ab} + \frac{f'\dot{a}}{fa}\dot{\phi} + \frac{f'\dot{b}}{fb}\dot{\phi} + \frac{f''}{f}\dot{\phi}^2 + \frac{\dot{\phi}^2}{4f} - \frac{V(\phi)}{2f} = 0 \quad (5)$$

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} - \frac{\ddot{\phi}}{2f'} - \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\frac{\dot{\phi}}{2f'} + \frac{1}{b^2} - \frac{V'(\phi)}{2f'} = 0 \quad (6)$$

$$\frac{\dot{b}^2}{b^2} + \frac{f'\dot{a}}{fa}\dot{\phi} + 2\frac{f'\dot{b}}{fb}\dot{\phi} - \frac{\dot{\phi}^2}{4f} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{1}{b^2} - \frac{V(\phi)}{2f} = 0 \quad (7)$$

where overdot and prime represent derivatives with respect to time and ϕ respectively. The Hessian determinant, $W = |\frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j}|$ turns out to be,

$$W = -16\pi f ab^4 (3f'^2 + f) \quad (8)$$

Hence, for $3f'^2 + f = 0$, whose exact solution is

$$f = -\frac{1}{12}(\phi - \phi_o)^2 \quad (9)$$

the Hessian determinant vanishes and the Lagrangian (3) becomes degenerate as in the Robertson-Walker case [8] and [9].

Let us now turn our attention to find the condition under which the Lagrangian(3) would admit Noether symmetry. In the Lagrangian under consideration the configuration space is $Q = (a, b, \phi)$, whose tangent space is $TQ = (a, b, \phi, \dot{a}, \dot{b}, \dot{\phi})$. Hence the infinitesimal generator of the Noether symmetry is

$$X = \alpha \frac{\partial}{\partial a} + \beta \frac{\partial}{\partial b} + \gamma \frac{\partial}{\partial \phi} + \dot{\alpha} \frac{\partial}{\partial \dot{a}} + \dot{\beta} \frac{\partial}{\partial \dot{b}} + \dot{\gamma} \frac{\partial}{\partial \dot{\phi}} \quad (10)$$

The existence of Noether symmetry implies the existence of a vector field X such that the Lie derivative of the Lagrangian with respect to the vector field vanishes, ie.

$$\mathcal{L}_X L = 0. \quad (11)$$

This yields an expression which is second degree in a, b and ϕ and whose coefficients are functions of a, b and ϕ only. Thus to satisfy equation [11], we obtain a set of following equations,

$$2f \frac{\partial \beta}{\partial a} + f' b \frac{\partial \gamma}{\partial a} = 0 \quad (12)$$

$$\alpha + 2b \frac{\partial \alpha}{\partial b} + 2a \frac{\partial \beta}{\partial b} + a \frac{f'}{f} (\gamma + 2b \frac{\partial \gamma}{\partial b}) = 0 \quad (13)$$

$$b\alpha + 2a\beta + 2ab \frac{\partial \gamma}{\partial \phi} - 4f' (b \frac{\partial \alpha}{\partial \phi} + 2a \frac{\partial \beta}{\partial \phi}) = 0 \quad (14)$$

$$\beta + b \frac{\partial \alpha}{\partial a} + a \frac{\partial \beta}{\partial a} + b \frac{\partial \beta}{\partial b} + b \frac{f'}{f} (\gamma + a \frac{\partial \gamma}{\partial a} + \frac{b}{2} \frac{\partial \gamma}{\partial b}) = 0 \quad (15)$$

$$f (b \frac{\partial \alpha}{\partial \phi} + a \frac{\partial \beta}{\partial \phi}) + f' (b\alpha + a\beta + \frac{b^2}{2} \frac{\partial \alpha}{\partial b} + ab \frac{\partial \beta}{\partial b} + ab \frac{\partial \gamma}{\partial \phi}) + f'' ab\gamma - \frac{ab^2}{4} \frac{\partial \gamma}{\partial b} = 0 \quad (16)$$

$$f \frac{\partial \beta}{\partial \phi} + f' (\beta + \frac{b}{2} \frac{\partial \alpha}{\partial a} + a \frac{\partial \beta}{\partial a} + \frac{b}{2} \frac{\partial \gamma}{\partial \phi}) + \frac{f'' b \gamma}{2} - \frac{ab}{4} \frac{\partial \gamma}{\partial a} = 0 \quad (17)$$

$$\alpha + \frac{f'}{f} a \gamma - \frac{ab^2}{2} [V(\frac{\alpha}{a} + 2\frac{\beta}{b}) + V' \gamma] = 0 \quad (18)$$

The above set of differential equations can essentially be solved by the method of separation of variables which finally yields a differential equation in f viz,

$$3f'^2 + f = o \quad (19)$$

whose solution is already given in equation (9). In addition α, β, γ and V are also obtained in the process as

$$\alpha = \frac{2l}{ab(\phi + \phi_0)^3}, \beta = \frac{l}{a^2(\phi + \phi_0)^3}, \gamma = -\frac{l}{a^2b(\phi + \phi_0)^2}, V = \lambda(\phi + \phi_0)^4 \quad (20)$$

where l, λ and ϕ_0 are constants of integration. So the Lagrangian (3) admits Noether symmetry under the above condition that f should have the form given by (9) while V should be quartic in the scalar field ϕ . However, it is to be noted that the form of f given by (9) makes the Lagrangian degenerate. Thus a constraint has been imposed on the Lagrangian in order that it admits Noether symmetry.

Now for Cartan one form

$$\theta_L = \frac{\partial L}{\partial \dot{a}} da + \frac{\partial L}{\partial \dot{b}} db + \frac{\partial L}{\partial \dot{\phi}} d\phi \quad (21)$$

the constant of motion $i_X \theta_L$ is obtained as,

$$F = \frac{\frac{d}{dt}[b(\phi + \phi_0)]}{a(\phi + \phi_0)^2} = \frac{\frac{d}{dt}(b\dot{\phi})}{a\dot{\phi}^2} (for \phi_0 = 0) \quad (22)$$

3 Analysing The Constraint And Presenting A Solution

The degeneracy in the Lagrangian imposed by the claim that it should have Noether symmetry, leads to constrained dynamics as mentioned in the introduction. This gives rise to underdetermined situation where the number of the true degrees of freedom exceeds the number of the field equations. To apprehend the situation, let us substitute f, f', f'' from equation (9) and V, V' from equation (20) in the field equations (4-7) to obtain

$$2\frac{\ddot{b}}{b} + 2\frac{\ddot{\phi}}{\phi} + \frac{\dot{b}^2}{b^2} - \frac{\dot{\phi}^2}{\phi^2} + 4\frac{\dot{b}\dot{\phi}}{b\phi} + \frac{1}{b^2} + 6\lambda\phi^2 = o \quad (23)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + 2\frac{\ddot{\phi}}{\phi} + \frac{\dot{a}\dot{b}}{ab} + 2\frac{\dot{a}\dot{\phi}}{a\phi} + 2\frac{\dot{b}\dot{\phi}}{b\phi} - \frac{\dot{\phi}^2}{\phi^2} + 6\lambda\phi^2 = o \quad (24)$$

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} + 3\frac{\ddot{\phi}}{\phi} + 2\frac{\dot{a}\dot{b}}{ab} + 3\frac{\dot{a}\dot{\phi}}{a\phi} + 6\frac{\dot{b}\dot{\phi}}{b\phi} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} + 12\lambda\phi^2 = o \quad (25)$$

$$\frac{\dot{b}^2}{b^2} + 3\frac{\dot{\phi}^2}{\phi^2} + 2\frac{\dot{a}\dot{\phi}}{a\phi} + 2\frac{\dot{a}\dot{b}}{ab} + 4\frac{\dot{b}\dot{\phi}}{b\phi} + \frac{1}{b^2} + 6\lambda b^2 = o \quad (26)$$

In addition we have yet another equation viz, equation (22), which is actually the constraint that has to be satisfied by the field equations (23-26). In order to see whether any new constraint arises from these field equations, we have to take time derivative of equation (22) and eliminate acceleration terms [10] between the equation thus obtained and the field equations (22-26). Time derivative of equation (22) is (using the same equation in it)

$$\frac{\ddot{b}}{b} + \frac{\ddot{\phi}}{\phi} = 2\frac{\dot{\phi}^2}{\phi^2} + F\frac{\dot{a}\dot{\phi}}{b} \quad (27)$$

Now eliminating acceleration terms between equations (23) and (27) one gets back the Hamiltonian constraint equation (26). Hence equation (13) is no longer an independent equation. In view of equations (22) and (26), one can obtain yet another constraint equation, viz,

$$\frac{d}{dt}(a\phi) = -\frac{1 + F^2 a^2 \phi^2}{2Fb} - \frac{3\lambda\phi^2 b}{F} \quad (28)$$

which can be used instead of equation (26). Differentiating equation (28) with respect to time and using the same equation once again in it, one obtains,

$$\frac{\ddot{a}}{a} + \frac{\ddot{\phi}}{\phi} = -2\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{1 + F^2 a^2 \phi^2}{2Fab^2\phi} \dot{b} - \frac{3\lambda\phi}{Fa} \dot{b} - \frac{6\lambda b}{Fa} \dot{\phi} + \frac{1 + F^2 a^2 \phi^2}{2b^2} + 3\lambda\phi^2 \quad (29)$$

In view of equations (27) and (29) one can now easily observe that equations (24) and (25) are trivially satisfied. Hence at this stage we are left with a pair of equations viz, equations (22) and (28) with three degrees of freedom viz, a, b and ϕ , leading to an underdetermined situation. This is the outcome of a degenerate Lagrangian. In order to obtain solution, one is now free to impose ‘one’ condition that would lead to physically acceptable solution. We are presenting here one such solution under the assumption,

$$a\phi = kb \quad (30)$$

where k is a constant and let it be positive definite ($k > 0$). In view of equation (30) equation (22) can immediately be integrated to yield,

$$b\phi = n \exp(Fkt) \quad (31)$$

where n is a constant of integration and considered to be positive definite ($n > 0$) too. Further for $\lambda = 0$, equation (28) can also be integrated in view of equation (30). the result is

$$b = \frac{1}{Fk} [m \exp(-Fkt) - 1]^{\frac{1}{2}} \quad (32)$$

where the overall negative sign has been chosen to reveal physically acceptable solution and m is yet another constant of integration which is considered to be greater than one ($m > 1$) for the same reason. Hence ϕ and a can also be obtained in view of equations (30), (31) and (32) as,

$$\phi = -nFk \frac{\exp(Fkt)}{[m \exp(-Fkt) - 1]^{\frac{1}{2}}}, a = \frac{\exp(-Fkt)}{nkF^2} [m \exp(-Fkt) - 1] \quad (33)$$

Now if one chooses $F = -c^2$, then the solutions (32) and (33) take the following form,

$$a = \frac{\exp(c^2kt)}{nkc^4} [m \exp(c^2kt) - 1], b = \frac{1}{kc^2} [m \exp(c^2kt) - 1]^{\frac{1}{2}}, \quad (34)$$

$$ab^2 = \frac{\exp(c^2kt)}{nk^3c^8} [m \exp(c^2kt) - 1]^2, \phi = nkc^2 \frac{\exp(-c^2kt)}{[m \exp(c^2kt) - 1]^{\frac{1}{2}}} \quad (35)$$

The above solution reveals that the universe admits inflation starting from a finite proper volume, under the choice of the constants already made viz, $k > 0, n > 0$ and $m > 1$. the scalar field at the initial epoch is finite and it falls off exponentially as the universe expands. The solution is singularity free although there is no question of graceful exit from inflation. the big-bang singularity is pushed back to the infinite past.

4 Some Other Symmetries In Kantowski-Sachs Model

In this section we shall first show that the whole laborious job that has been carried out in the preceeding section, to find the set of equations (12) to (18) and to solve them by the method of the separation of variables to obtain conditions under which the Lagrangian (3) admits Noether symmetry, is not at all required. Rather we can construct a pair of equations from the set of field equations (4) to (7), one of which can immediately extract the conditions for which the Lagrangian would admit dynamical symmetry and the other can find the corresponding conserved current. We shall further show that one of the dynamical symmetries obtained in the process is of Noether class and there exists dynamical symmetries of some other type that we could not obtain by applying Noether’s theorem in the preceeding section.

The first one of this pair is the continuity equation. This equation is obtained by eliminating \ddot{a} and \ddot{b} from the field equations (4) to (6) and then comparing it with the Hamiltonian constraint equation (7). The equation thus formed is,

$$2(3f'^2 + f)(\ddot{\phi} + \frac{\dot{a}}{a}\dot{\phi} + 2\frac{\dot{b}}{b}\dot{\phi}) + f'(6f'' + 1)\dot{\phi}^2 + 2(fV' - 2Vf') = 0. \quad (36)$$

All dynamical symmetries are hidden in this equation. To find Noether symmetry one has to choose f and V in such a way that equation (36) is satisfied identically. The choice is quite trivial viz, the coefficients of the derivatives of ϕ, a and b should vanish separately. This implies $3f'^2 + f = 0$ ie, $f = -\frac{1}{12}\phi^2$ and as such $6f'' + 1 = 0$ too. From the last term of equation (36) one can see that V is proportional to f^2 and hence is in the form $V = \lambda\phi^4$. These results are already obtained in equations (19) and (20) of the preceeding section. To obtain the conserved

current we construct yet another equation and that is done simply by eliminating terms in the field equations which are free from time derivatives viz, $\frac{1}{b^2}, V(\phi)$ and $V'(\phi)$ in the present context. This is done by taking the difference of equations (1) and (4), which yields,

$$2\frac{\ddot{b}}{b} + \frac{f'}{f}\ddot{\phi} + \left(\frac{2f''+1}{2f}\right)\dot{\phi}^2 - \frac{f'\dot{a}}{fa}\dot{\phi} - 2\frac{\dot{a}\dot{b}}{ab} = 0 \quad (37)$$

This equation in view of the solution of f obtained from equation (36), reads

$$\frac{\ddot{b}}{b} + \frac{\ddot{\phi}}{\phi} - 2\frac{\dot{\phi}^2}{\phi^2} - \frac{\dot{a}\dot{\phi}}{a\phi} - \frac{\dot{a}\dot{b}}{ab} = 0 \quad (38)$$

whose first integral yields the conserved current obtained in equation (22). Thus, we have shown that the Noether symmetry can even be obtained in view of the continuity equation (36) and the corresponding conserved current from equation (37), without invoking equations (11) and (21).

Let us now proceed to find some other type of dynamical symmetry that we did not find in the preceeding section, for which we choose f in the form

$$f = \epsilon\phi^2 \quad (39)$$

Further we choose the potential in the form

$$V = \lambda\phi^4 \quad (40)$$

then equation (36) can be written as

$$(12\epsilon + 1)\left(\ddot{\phi} + \frac{\dot{a}}{a}\dot{\phi} + 2\frac{\dot{b}}{b}\dot{\phi} + \frac{\dot{\phi}^2}{\phi}\right) = 0 \quad (41)$$

For $\epsilon = -1/12$, we regain Noether symmetry. However, for any arbitrary ϵ other than zero or $(-1/12)$, equation (41) leads to

$$\frac{\ddot{\phi}}{\phi} + \frac{\dot{a}\dot{\phi}}{a\phi} + 2\frac{\dot{b}\dot{\phi}}{b\phi} + \frac{\dot{\phi}^2}{\phi^2} = 0 \quad (42)$$

whose first integral is

$$ab^2\phi\dot{\phi} = \text{constant} \quad (43)$$

Thus we obtain yet another dynamical symmetry of the system for arbitrary value of ϵ , for which the conserved current is given by equation (43). It is to be noted that the existence of this dynamical symmetry does not make the Lagrangian degenerate, since the Hessian determinant given by equation (8) does not turn out to be zero. Since this symmetry has not been obtained by the application of Noether's theorem, therefore we conclude that not all the dynamical symmetries of a Lagrangian are of Noether class. The dynamical symmetry thus obtained here is of the same form that we have already seen in connection with the Robertson-Walker metric [9]. This type of symmetry is of much interest. Since the symmetry exists for arbitrary ϵ , it exists for conformally coupled scalar field $\epsilon = 1/6$ as well. Further as already mentioned in the introduction that large negative value of ϵ might give rise to well behaved self consistent classical solution admitting inflation [3] on one hand and can produce density perturbations of amplitude consistent with the large scale behavior keeping the cosmological constant λ within the ordinary GUT range, on the other, therefore the existence of dynamical symmetry corresponding to arbitrary value of ϵ is definitely of much importance.

5 Concluding Remarks

In a recent communication we have come across an important and wonderful result, while reviewing the works of Capozziello et al [8] in connection with the Noether symmetry in the Robertson-Walker metric. The result is that, even if there exists certain forms of $f(\phi)$ and $V(\phi)$ and hence a vector field X such that $\mathcal{L}_X L = 0$: the form of $V(\phi)$ might not satisfy the field equations. We have not come across such a result earlier and as such do not know the reason as yet. However, we have observed that only one, viz, the continuity equation suffice to check wheather the field equations are satisfied or not. In the context of the induced theory of gravity, this equation turned out to be a very important one to explore all sorts of existing dynamical symmetries of a Lagrangian.

In the present paper, we have shown that the Noether symmetry of the induced theory of gravity in the Kantowski-Sachs model can be traced from the continuity equation, while instead of using the Cartan's one form, the conserved current can simply be found from equation (37) or (38), which is obtained from yet another combination of the field equations. Though the Noether symmetry makes the Lagrangian degenerate and hence introduces a constraint which reduces the number of independent field equations to the number of true degrees of freedom by one, causing underdeterminacy, yet the field equations are found to admit inflationary solution under a suitable assumption. This result definitely proves that the conclusion made by Capozziello et al [8], viz, degeneracy leads to trivial solutions, is wrong.

It has been further observed that all sorts of dynamical symmetries of a Lagrangian can be explored from the continuity equation only, at least in induced theory of gravity. This equation further reveals dynamical symmetries other than the Noether symmetry, for a Lagrangian. This confirms that not all the dynamical symmetries of a system belong to the Noether class.

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